CHAPTER # 1 INTRODUCTION

1. Magnetic Circuits

The permanent magnet shown in Fig. 1, has a flux lines whose direction is indicated using a compass needle. In the upper half of the figure, the S end of the diamond-shaped compass settles closest to the N pole of the magnet, while in the lower half of the figure, the N end of the compass seeks the S of the magnet. This immediately suggests that there is a direction associated with the lines of flux, as shown by the arrows on the flux lines, which conventionally are taken as positively directed from the N to the S pole of the bar magnet.

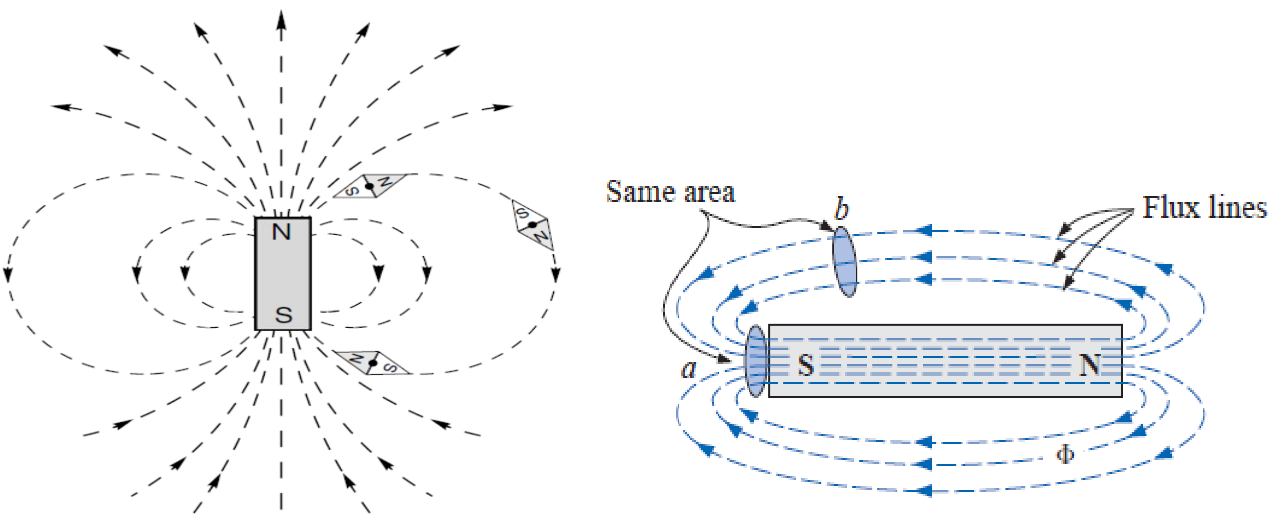


Fig. 1 Magnetic flux lines produced by a permanent magnet

Magnetic flux lines do not have origins or terminating points as do electric flux lines but exist in continuous loops, as shown in Fig. 1. The symbol for magnetic flux is the Greek letter Φ (phi). The magnetic field strength at point *a* is twice that at *b* since twice as many magnetic flux lines are associated with the perpendicular plane at *a* than at b. this means, the strength of permanent magnets is always stronger near the poles.
The continuous magnetic flux line will strive to occupy as small an area as possible. If unlike poles of two permanent magnets are brought together, the magnets will attract, and the flux distribution will be as shown in Fig. 2.

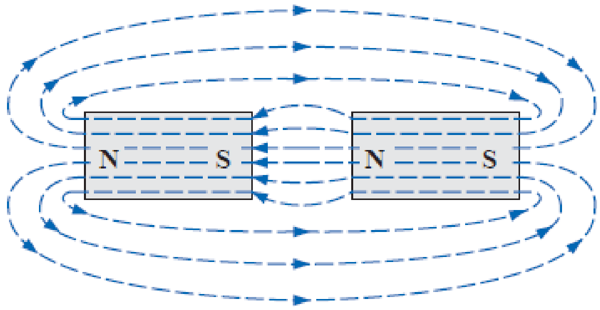


Fig. 2, Flux distribution for two adjacent, opposite poles.

If like poles are brought together, the magnets will repel, and the flux distribution will be as shown in Fig. 3.

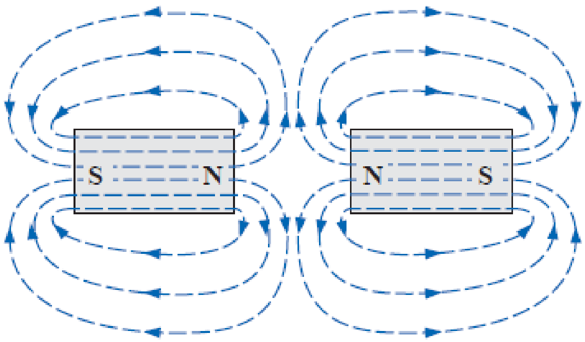


Fig. 3, Flux distribution for two adjacent, like poles.

A magnetic field (represented by concentric magnetic flux lines, as in Fig. 4) is present around every wire that carries an electric current. The direction of the magnetic flux lines can be found simply by placing the thumb of the *right* hand in the direction of current flow and noting the direction of the fingers. This method is commonly called the *right-hand rule.*

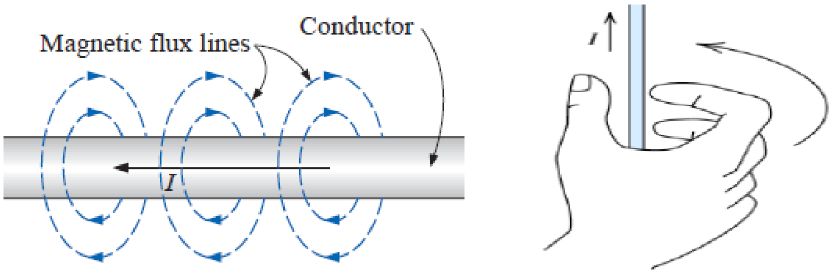


Fig. 4, Magnetic flux lines around a current-carrying conductor.

If the conductor is wound in a single-turn coil (Fig. 5), the resulting flux will flow in a common direction through the center of the coil.

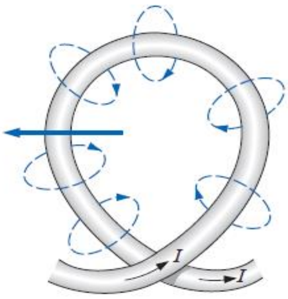


Fig. 5, Flux distribution of a single-turn coil.

A coil of more than one turn would produce a magnetic field that would exist in a continuous path through and around the coil (Fig. 6). The flux distribution of the coil is quite similar to that of the permanent magnet, and this is called electromagnet.

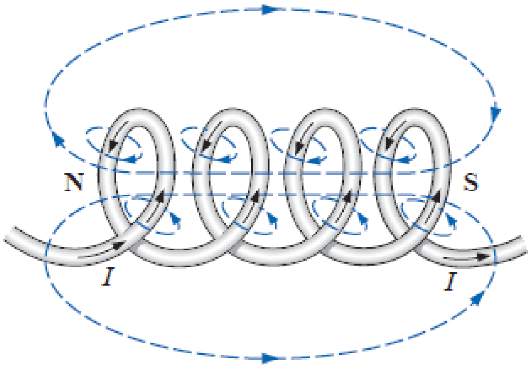


Fig. 6, Flux distribution of a current-carrying coil.

The direction of flux lines can be determined for the electromagnet by placing the fingers of the right hand in the direction of current flow around the core. The thumb will then point in the direction of the north pole of the induced magnetic flux, as demonstrated in Fig. 7. The cross and dot refer to the tail and head of the arrow, respectively.

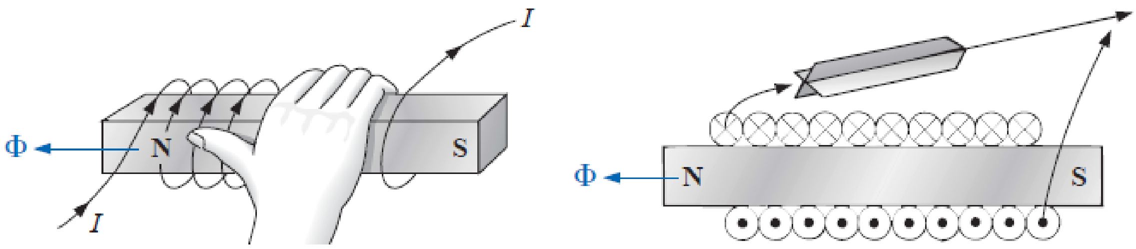


Fig. 7, Determining the direction of flux for an electromagnet

The areas of application for electromagnetic effects are shown in Fig. 8.

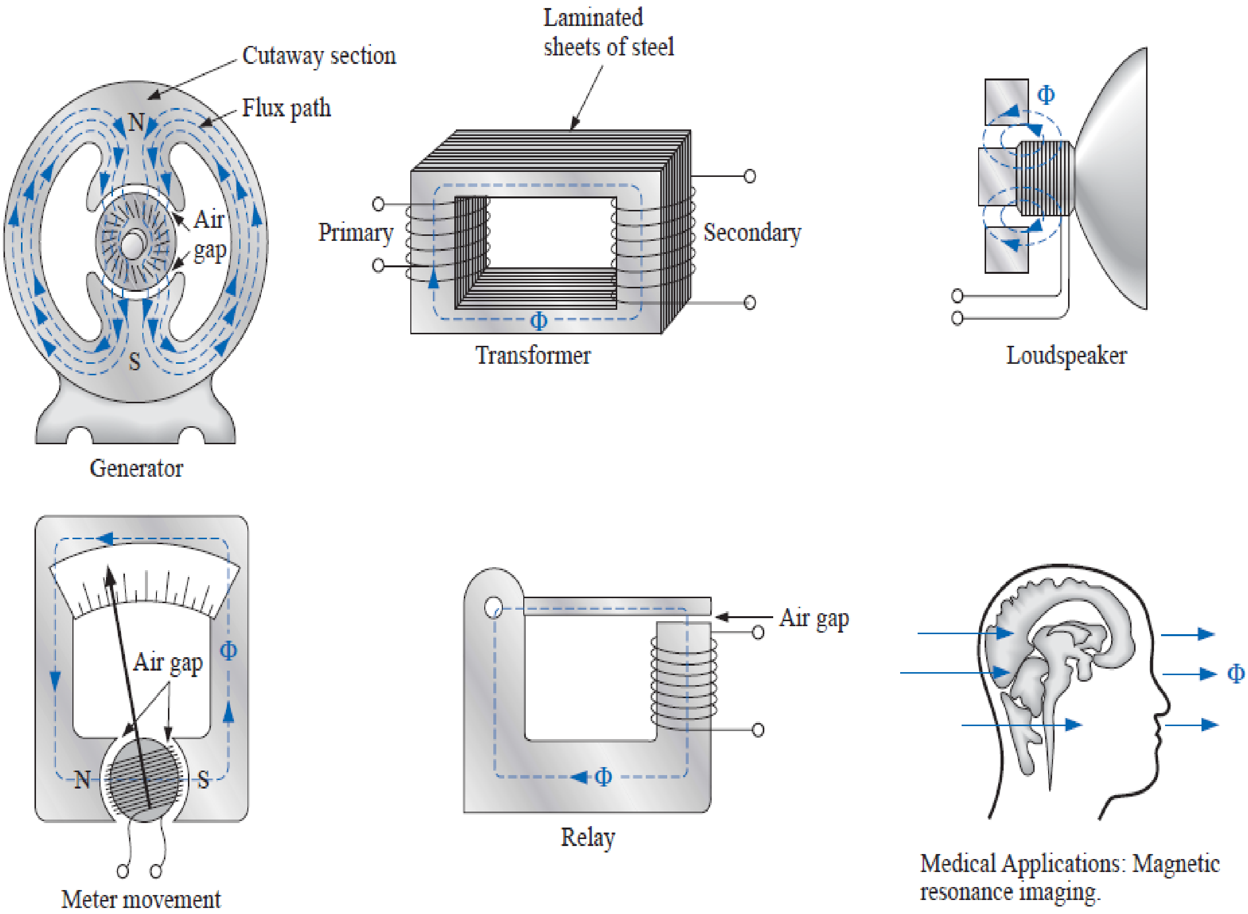


Fig. 8, Some areas of application of magnetic effects.

In the SI system of units, magnetic flux is measured in *Webers*. The number of flux lines per unit area is called the **flux density,** is denoted by the capital letter *B*, and is measured in *teslas*. Its magnitude is determined by the following equation:

$$B = \frac{\Phi}{A}$$

B = teslas (T)
Φ = webers (Wb)
A = square meters (m^2)

To illustrate the meaning of flux density, consider part of magnetic circuit shown in Fig. 9. The cross-sectional area at bb' is twice that at aa', but the number of flux lines is constant so the flux density at bb' is half that at aa'.

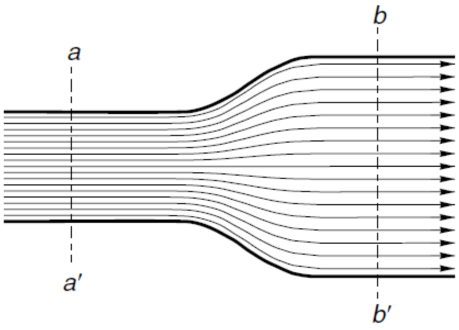


Fig. 9 Magnetic flux lines inside part of an iron magnetic circuit

But the number of flux lines is denoted by the magnetic field intensity (H), which is constant either at aa' or at bb'. The relationship between current (i) and field intensity (H) can be obtained from **Ampere Law** which states that the line integral of the magnetic field intensity H around a closed path is equal to the total current linked by the contour, as shown in Fig. 10.

$$\oint \mathbf{H} \cdot \mathbf{dl} = \sum i = i_1 + i_2 - i_3$$

Or

$$\oint H \, dl \cos\theta = \sum i$$

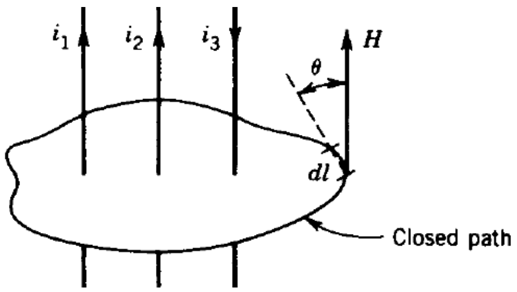


Fig. 10, Illustration of Ampere law

- Faraday's law:

  A moving conductor cutting the lines of force (flux) of a constant magnetic field has a voltage induced in it.

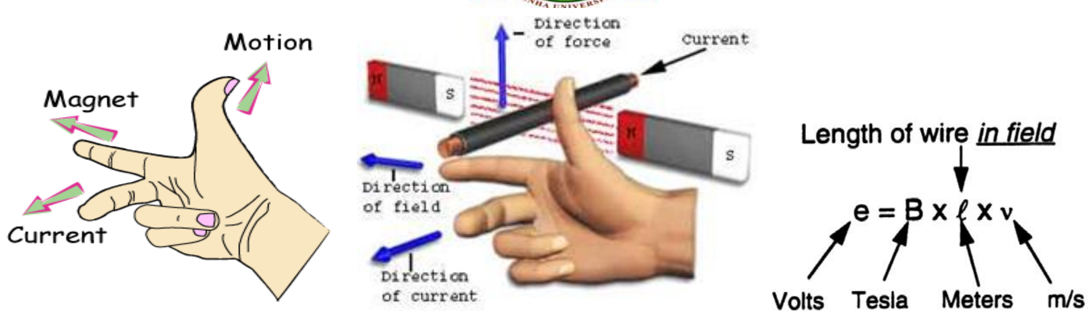


Fig. 11, Fleming right hand rule

Fleming right hand rule is used to determine the direction of the induced emf. If the thumb finger represents the force (F) and the forefinger represents the flux (B), then middle finger represents the induced voltage (*generator action*)

- Ampere-Biot-Savart law in its simplest form can be seen as the "reverse" of Faraday's law. While Faraday predicts a voltage induced in a conductor moving across a magnetic field, Ampere-Biot-Savart law establishes that a force is generated on a current carrying conductor located in a magnetic field.

Fleming Left-hand rule is used to determine the direction of the mechanical force. If the middle finger represents the current (I) and the forefinger represents the flux (B), then thumb represents the force (F) (*motor action*)

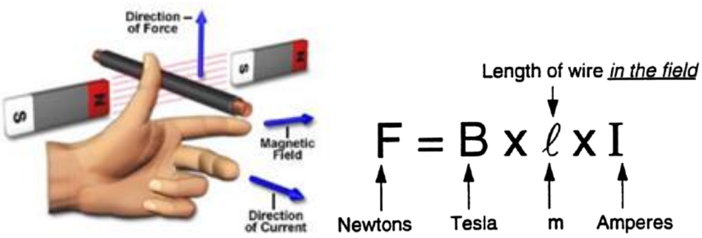


Fig. 12, Fleming left hand rule

Magnetic fields are the **fundamental mechanism by which energy is converted** from one form to another in motors, generators and transformers.

The magnetic field intensity (H) produces a magnetic flux density (B) everywhere it exists. These quantities are functionally related by:

$$B = \mu H$$

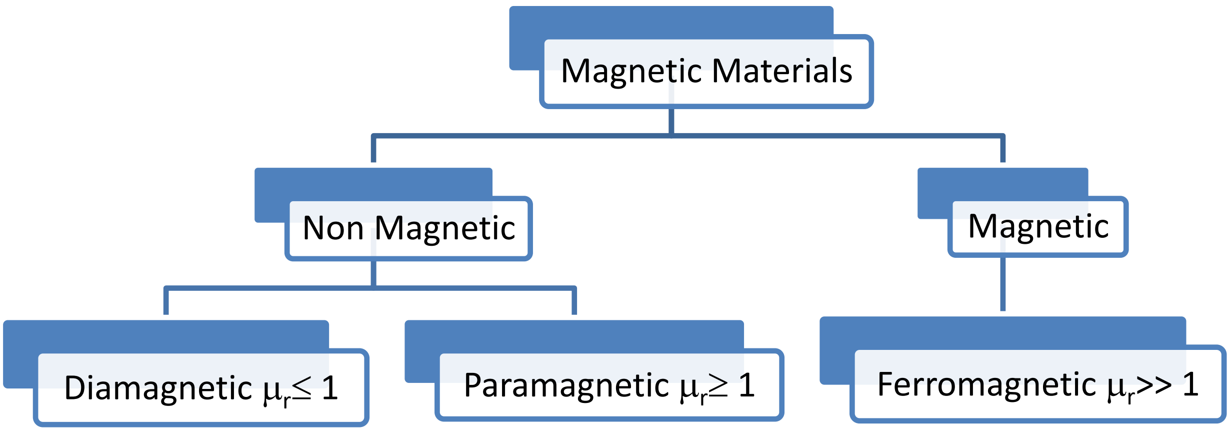
The **permeability نفاذية**(μ) of a material, therefore, is a measure of the ease with which magnetic flux lines can be established in the material. It is similar in many respects to conductivity in electric circuits. The permeability of free space μ_o (vacuum) is:

$$\mu_o = 4\pi \times 10^{-7} \quad \frac{Wb}{At.m}$$

The ratio of the permeability of a material to that of free space is called its **relative permeability** (μ_r)**;** that is,

$$\mu_r = \frac{\mu}{\mu_o}$$

According to the relative permeability, magnetic materials are classified to:



Materials that have permeabilities slightly less than that of free space are said to be **diamagnetic.** It is due to the non-cooperative behavior of orbiting electrons when exposed to an applied magnetic field. Diamagnetic substances are composed of atoms which have no net magnetic moments. However, when exposed to a field, a negative magnetization is produced and thus the susceptibility is negative (repelled by magnetic field). Example is Graphite

Materials with permeabilities slightly greater than that of free space are said to be paramagnetic. Some of the atoms in the material have a net magnetic moment. However, the individual magnetic moments do not interact magnetically, and like diamagnetism, the magnetization is zero when the field is removed. In the presence of a field, there is now a partial alignment of the atomic magnetic moments in the direction of the field, resulting in a net positive magnetization and positive susceptibility. (attracted by magnetic field). Example is Aluminum.

Magnetic materials, such as iron, nickel, steel, cobalt, and alloys of these metals, have permeabilities hundreds and even thousands of times that of free space. Materials with these very high permeabilities are referred to as **ferromagnetic.**

The resistance of a material to the flow of charge (current) is determined for electric circuits by the equation

$$R = \rho \frac{l}{A} \qquad \text{(ohms, } \Omega\text{)}$$

The reluctance of a material to the setting up of magnetic flux lines in the material is determined by the following equation:

$$\mathscr{R} = \frac{l}{\mu A} \qquad \text{(rels, or At/Wb)}$$

where $\Re$ is the reluctance, *l* is the length of the magnetic path, and A is the cross-sectional area. Note that the reluctance is inversely proportional to the area and directly proportional to the length. The reluctance, however, is inversely proportional to the permeability. Obviously, therefore, materials with high permeability, such as the ferromagnetics, have very small reluctances and will result in an increased measure of flux through the core.

The Ohm's low of magnetic circuits is given by:

$$\Phi = \frac{\mathscr{F}}{\mathscr{R}}$$

The magnetomotive force MMF (F) is proportional to the product of the number of turns around the core (in which the flux is to be established) and the current through the turns of wire (Fig.13). In equation form, $\mathscr{F} = NI$

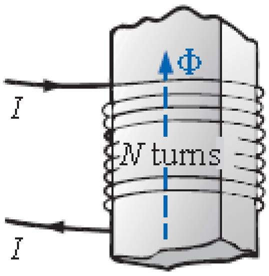


Fig. 13, Defining the components of a magnetomotive force.

The magnetomotive force per unit length is called the magnetizing force (H). In equation form,

$$H = \frac{\mathscr{F}}{l} \qquad \text{(At/m)}$$

Substituting with the value of the MMF,

$$H = \frac{NI}{l} \qquad \text{(At/m)}$$

For the magnetic circuit shown in Fig. 14, if NI=40 At and the length of the core = 0.2 m, then the magnetizing force H = 40/0.2 = 200 At/m

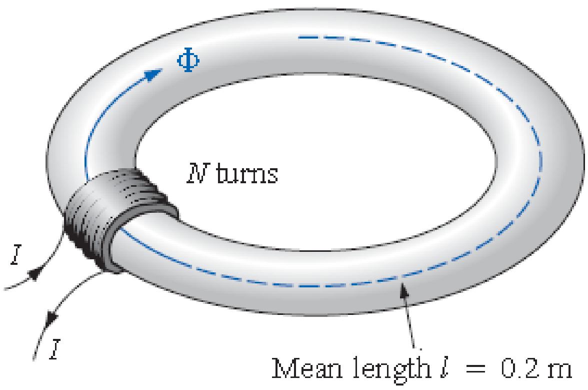


Fig. 14, Defining the magnetizing force of a magnetic circuit.

The magnetizing force also depend on the core material, therefore,

$$B = \mu H$$

Where B is the magnetic flux density Wb/m^2 (Tesla)

Substituting H by its value,

$$B = \mu \frac{Ni}{l_c}$$

But the flux (ϕ) produced in the = B×A, then

$$\phi = Ni\frac{\mu A}{l_c}$$

The flow of magnetic flux induced in the ferromagnetic core can be made analogous to an electrical circuit, as shown in Fig. 15, hence the name magnetic circuit is used.

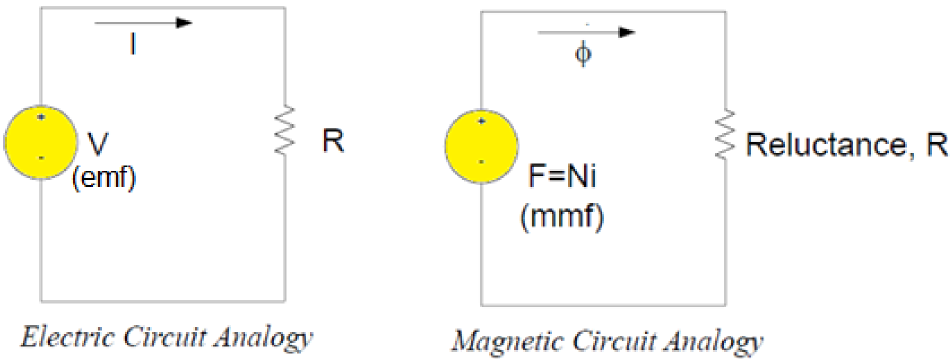


Fig. 15, Magnetic circuit analogy

Referring to the magnetic circuit analogy, F is denoted as magnetomotive force (mmf) which is similar to electromotive force in an electrical circuit (emf). Therefore, we can safely say that F is the prime mover or force which pushes magnetic flux around a ferromagnetic core at a value of Ni (refer to ampere's law).

Since V = IR (electric circuit), then F=ϕ ℜ

$$Ni = \phi\frac{l_c}{\mu A}$$

Then the reluctance (ℜ) is defined as:

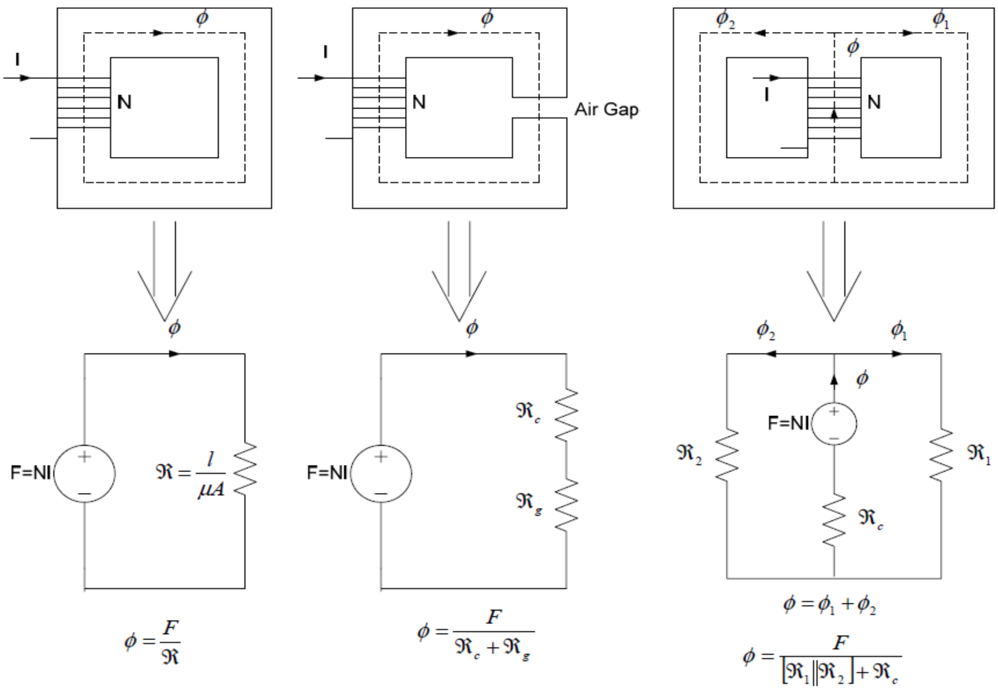
$$\Re = \frac{l_c}{\mu A}$$

ELECTRIC CIRCUIT	***MAGNETIC CIRCUIT***
E = EMF	F = MMF
R = Resistance	ℜ = Reluctance
I = Current	ϕ = Flux
σ = Conductivity	μ = Permeability
$E = RI$	$mmf = \Re\phi$
$R = \frac{l}{\sigma A} = \frac{\rho l}{A}$	$\Re = \frac{l}{\mu A}$
$R_{series} = R_1 + R_2 + ... + R_N$	$\Re_{series} = \Re_1 + \Re_2 + ... + \Re_N$
$1/R_{parallel} = 1/R_1 + 1/R_2 + ... + 1/R_N$	$1/\Re_{parallel} = 1/\Re_1 + 1/\Re_2 + ... + 1/\Re_N$

In order to analyze any magnetic circuit, two steps are mandatory as illustrated by figure given below.

Step #1: Find the electric equivalent circuit that represents the magnetic circuit.

Step #2: Analyze the electric circuit to solve for the magnetic circuit quantities.

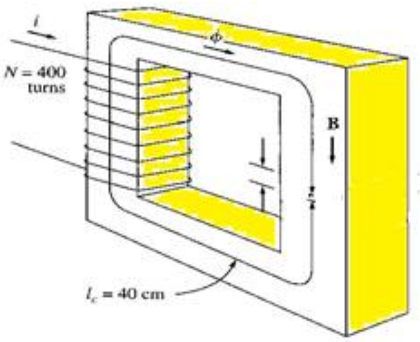


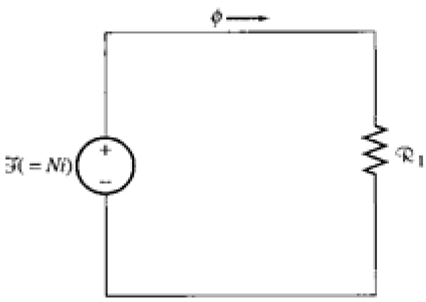
Example (1):

A ferromagnetic core with mean path length is 40cm. The CSA of the core is 12cm^2, the relative permeability of the core is 4000, and the coil of wire on the core has 400 turns. Find:

(a) The reluctance of the flux path,

(b) The current required to produce a flux density of 0.5T in the core.





The analogous circuit of the core is represented as shown above where reluctance R_1 represent the core

$$R_1 = \frac{40 \times 10^{-2}}{4000 \times 4\pi \times 10^{-7} \times 12 \times 10^{-4}} = 66314.5596\ At/Wb$$

$\Phi = B \times A = 0.5 \times 12\times10^{-4} = 6\times10^{-4}$ Wb

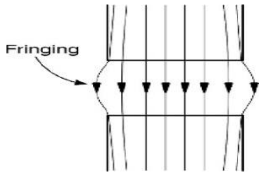
$N\times I = \Phi \times R_{total} = 6\times10^{-4} \times 66314.5596 = 39.7887$ At

$I = 39.7887 / 400 = 99.4718$ mA

The fringing effect results from the presence of the air gap in the magnetic circuit. The main consequence of the fringing effect is to make the magnetic flux density of the air gap (B_g) different from the flux density of the core (B_c) due to the path of the flux.

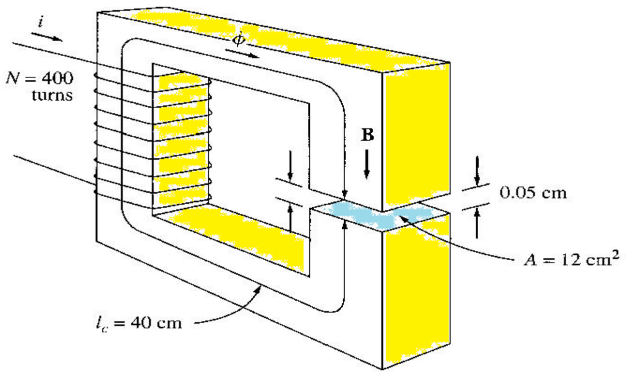
$$\phi_c = \phi_g ,$$

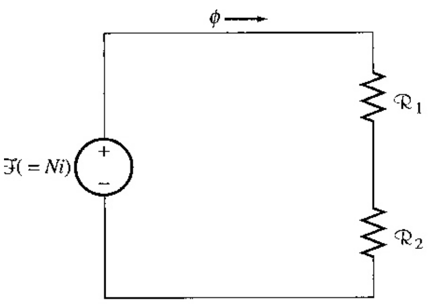
but $$B_c \neq B_g$$



Example (2)

A ferromagnetic core given in Example (1). There is a small gap of 0.05cm in the structure of the otherwise whole core. Assume that fringing effect is neglected. Find:
(a) The total reluctance of the flux path (iron plus air gap)
(b) The current required to produce a flux density of 0.5T in the air gap.





The analogous circuit of the core is represented as shown above where reluctance R_1 represent the core and R_2 represent the air gap.

$$R_1 = \frac{40 \times 10^{-2}}{4000 \times 4\pi \times 10^{-7} \times 12 \times 10^{-4}} = 66314.5596 \quad At/Wb$$

$$R_2 = \frac{0.05 \times 10^{-2}}{1 \times 4\pi \times 10^{-7} \times 12 \times 10^{-4}} = 331572.7981 \quad At/Wb$$

The two reluctances are connected in series

$R_{total} = R_1 + R_2 = 397887.3577$ At/Wb ##

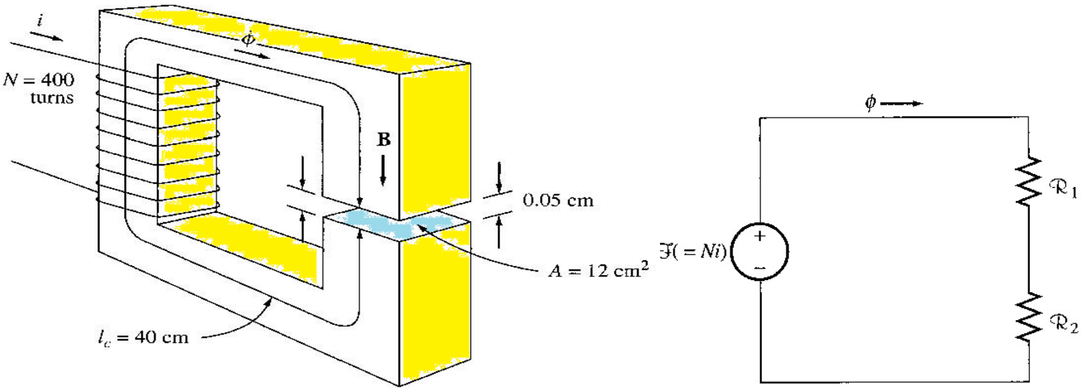
$\Phi = B \times A_g = 0.5 \times12\times10^{-4} = 6\times10^{-4}$ Wb

$N\times I = \Phi \times R_{total} = 6\times10^{-4} \times 397887.3577 = 238.7324$ At

I = 238.7324 / 400 = 0.5968 A

Example (3)

A ferromagnetic core given in Example (2). Assume that fringing effect in the air gap increases the effective CSA of the gap by 5%. Find:
(a) The total reluctance of the flux path (iron plus air gap)
(b) The current required to produce a flux density of 0.5T in the air gap.



The analogous circuit of the core is represented as shown above where reluctance R_1 represent the core and R_2 represent the air gap.

$$R_1 = \frac{40 \times 10^{-2}}{4000 \times 4\pi \times 10^{-7} \times 12 \times 10^{-4}} = 66314.5596 \quad At/Wb$$

$$R_2 = \frac{0.05 \times 10^{-2}}{1 \times 4\pi \times 10^{-7} \times 1.05 \times 12 \times 10^{-4}} = 315783.6172 \quad At/Wb$$

The two reluctances are connected in series

$R_{total} = R_1 + R_2 = 382098.1768$ At/Wb ##

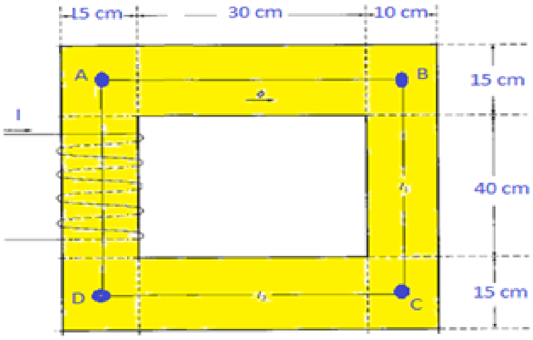
$\Phi = B \times A_g = 0.5 \times 1.05 \times 12\times10^{-4} = 6.3\times10^{-4}$ Wb

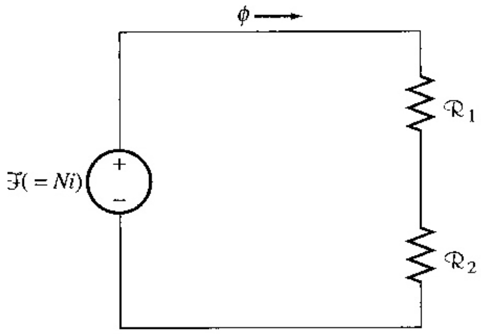
$N\times I = \Phi \times R_{total} = 6.3\times10^{-4} \times 382098.1768 = 240.7219$ At

I = 240.7219 / 400 = 0.6018 A

Example (4):

A ferromagnetic core is shown. Three sides of this core are of uniform width, while the fourth side is somewhat thinner. The depth of the core (into the page) is 10cm, and the other dimensions are shown in the figure. There is a 200 turn coil wrapped around the left side of the core. Assuming relative permeability μ_r of 2500, how much flux will be produced by a 1A input current?





Calculate length of each section:

$L_{AB}=L_{CD}$ = 7.5+30+5 = 42.5 cm

$L_{BC}=L_{DA}$ = 7.5+40+7.5 = 55 cm = 0.55 m

L_{CDAB} = 42.5 + 55 + 42.5 = 140 cm = 1.4 m

Calculate the reluctance:

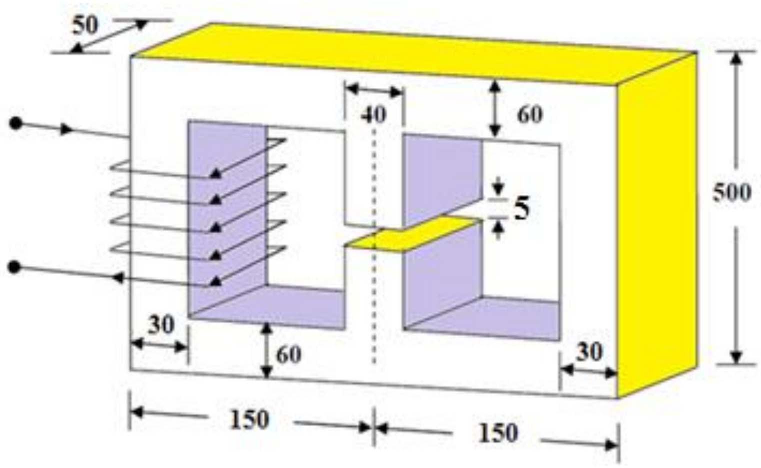
$$\Re_{CDAB} = \frac{l_{CDAB}}{\mu\, A_1} = \frac{1.4}{2500 \times 4\pi \times 10^{-7} \times 15 \times 10 \times 10^{-4}} = 29708.92271$$

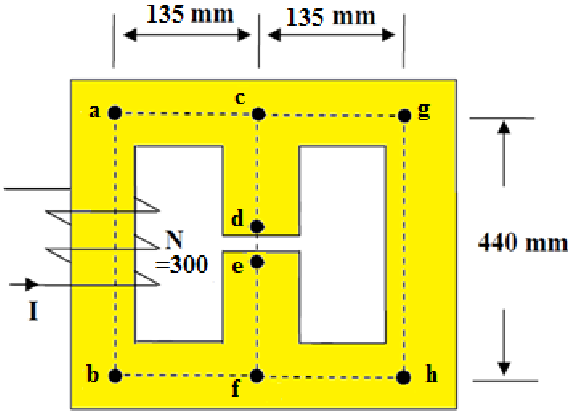
$$\Re_{BC} = \frac{l_{BC}}{\mu\, A_2} = \frac{0.55}{2500 \times 4\pi \times 10^{-7} \times 10 \times 10 \times 10^{-4}} = 17507.04374$$

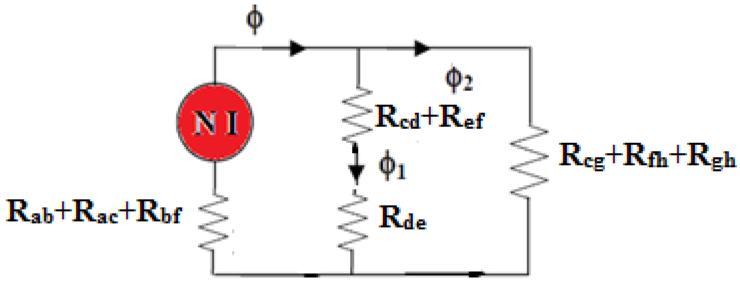
$200 \times 1 = \Phi \times (\Re_{CDAB} + \Re_{BC}) \rightarrow \Phi = 4.236$ m Wb.

Example (5):

In the magnetic circuit shown below with all dimensions in mm, calculate the required current to be passed in the coil having 300 turns in order to establish a flux of 2.28 mWb in the air gap. Consider the fringing effect at the air gap by 7% and the relative permeability of the core is 4000







$$R_{ab} = R_{gh} = \frac{440 \times 10^{-3}}{4000 \times 4\pi \times 10^{-7} \times 30 \times 50 \times 10^{-6}} = 58356.81247$$

$$R_{bf} = R_{ac} = R_{cg} = R_{fh} = \frac{135 \times 10^{-3}}{4000 \times 4\pi \times 10^{-7} \times 60 \times 50 \times 10^{-6}} = 8952.465549$$

$$R_{cd} = R_{ef} = \frac{217.5 \times 10^{-3}}{4000 \times 4\pi \times 10^{-7} \times 40 \times 50 \times 10^{-6}} = 21635.1251$$

$$R_{de} = \frac{5 \times 10^{-3}}{4\pi \times 10^{-7} \times (40 \times 50 \times 10^{-6}) \times 1.07} = 1859286.718$$

$$MMF_1 = \varphi_1(R_{ef} + R_{cd} + R_{de}) = 2.28 \times 10^{-3} \times 1902556.968 = 4337.823$$

$$MMF_2 = \varphi_2(R_{cg} + R_{fh} + R_{gh}) = 4337.823$$

$$\varphi_2 = 56.881 \times 10^{-3}\ Wb$$

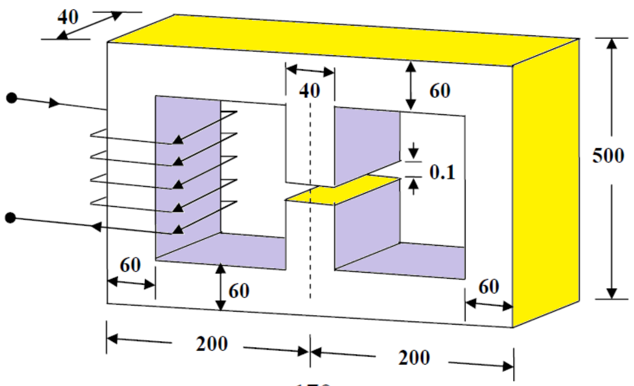
$$\varphi = \varphi_1 + \varphi_2 = (2.28 + 56.881) \times 10^{-3} = 59.161 \times 10^{-3}\ Wb$$

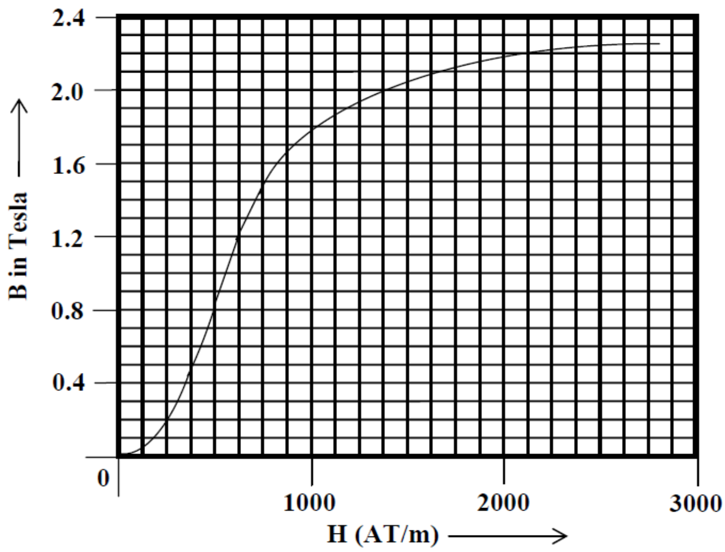
$$N \times i = \varphi(R_{ab} + R_{ac} + R_{bf}) + MMF_1 = 59.161 \times 10^{-3} \times 76261.74357 + 4337.823$$

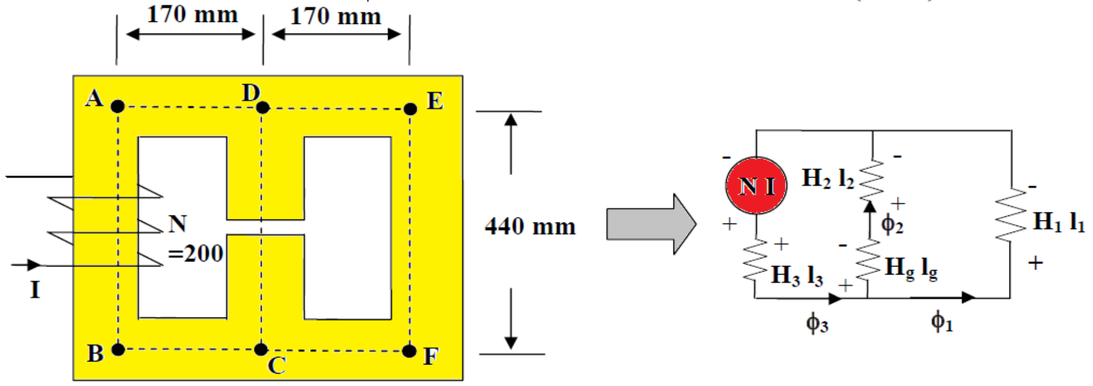
$$i = \frac{8849.544}{300} = 29.4985\ A$$

Example (6):

In the magnetic circuit shown in Figure below with all dimensions in mm, calculate the required current to be passed in the coil having 200 turns in order to establish a flux of 1.28 mWb in the air gap. Neglect fringing effect and leakage flux. The B-H curve of the material is given. Permeability of air may be taken as, $\mu_0 = 4\pi \times 10^{-7}$ H/m







$$\phi_g = \phi_2 \quad = \quad 1.28 \times 10^{-3}$$

Cross sectional area of central limb $A_2 \quad = \quad 16 \times 10\text{-}4\ \text{m}^2$

$$\text{Flux density } B_g = B_2 \quad = \quad \frac{1.28\times10^{-3}}{16\times10^{-4}}\,\text{T} \quad = \quad 0.8\ \text{T}$$

$$\therefore, H_g = \frac{B_g}{\mu_0} \quad = \quad \frac{0.8}{4\pi\times10^{-7}}\,\text{AT/m}$$

$$= \quad 63.66 \times 10^4\ \text{AT/m}$$

mmf required for gap $H_g l_g \quad = \quad 63.66 \times 10^4 \times 1 \times 10^{-4}\ \text{AT} \quad = \quad 63.66\ \text{AT}$

Now we must calculate the mmf required in the iron portion of the central limb as follows:

flux density, $B_2 \quad = \quad 0.8$ T $\because$ fringing & leakage neglected

corresponding H from graph, $H_2 \quad \approx \quad 500$ AT /m

Mean iron length, $l_2 \quad = \quad (440 - 0.1)$ mm $\quad \approx \quad 0.44$ m

mmf required for iron portion, $H_2 l_2 \quad = \quad 220$ AT

Total mmf required for iron & air gap, $\quad = \quad (220 + 63.66)$ AT

$mmf_{CD} \quad = \quad 283.66$ AT.

Due to parallel connection, mmf acting across path 1 is same as mmf acting across path 2.
Our intention here, will be to calculate ϕ_1 in path 1.

$$\text{mean length of the path, } l_1 = l_{DE} + l_{EF} + l_{FC} = 2 \times 170 + 440 \text{ mm} = 0.78 \text{ m}$$

$$\because H_1 = \frac{283.66}{0.78} = 363.67 \text{ AT/m}$$

$$\text{corresponding flux density from graph, } B_1 \approx 0.39 \text{ T}$$

$$\therefore \text{flux, } \phi_1 = B_1 A_1 = 0.39 \times 24 \times 10^{-4} = 0.94 \times 10^{-3} \text{ Wb}$$

we calculate the mmf necessary to drive ϕ_3 in path 3 as follows.

$$\text{flux in path 3, } \phi_3 = \phi_1 + \phi_2 = 2.22 \times 10^{-3} \text{ Wb}$$

$$\text{flux density, } B_3 = \frac{\phi_3}{A_3} = \frac{2.22 \times 10^{-3}}{24 \times 10^{-4}}$$

$$\therefore B_3 = 0.925 \text{ T}$$

$$\text{corresponding H from graph, } H_3 \approx 562.5 \text{ AT/m}$$

$$\text{mean length of path 3, } l_3 = 2 \times 170 + 440 \text{ mm} = 0.78 \text{ m}$$

$$\text{total mmf required for path 3} = H_3 l_3 = 562.5 \times 0.78 = 438.7 \text{ AT}$$

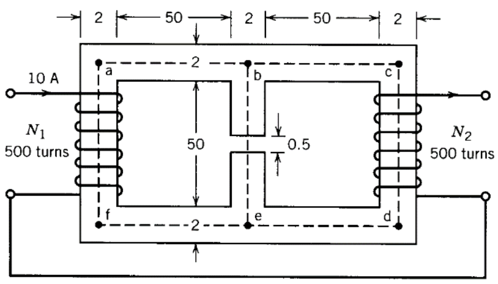
$$\therefore \text{mmf to be supplied by the coil, } NI = 283.66 + 438.7 \text{ AT}$$

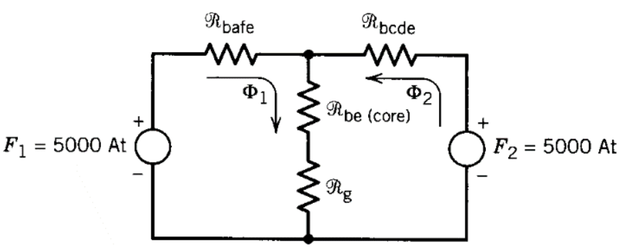
$$\text{or } 200I = 722.36 \text{ AT}$$

$$\therefore \text{exciting current needed, } I = \frac{722.36}{200} \text{ A} = 3.61 \text{ A}$$

Example (7):

For the magnetic circuit shown below, the relative permeability of the ferromagnetic material of the core is 1200. Neglect leakage and fringing. All dimensions are in cm, and the magnetic material has a square CSA. Determine the airgap flux and the magnetic field intensity in the airgap.





$F_1 = N_1 I_1 = 500 \times 10 = 5000$ At $\qquad F_2 = N_2 I_2 = 500 \times 10 = 5000$ At

$$\mu_c = 1200\mu_0 = 1200 \times 4\pi 10^{-7}$$

$$\mathcal{R}_{bafe} = \frac{l_{bafe}}{\mu_c A_c} = \frac{3 \times 52 \times 10^{-2}}{1200 \times 4\pi 10^{-7} \times 4 \times 10^{-4}} = 2.58 \times 10^6 \text{ At/Wb}$$

From symmetry $\mathcal{R}_{bcde} = \mathcal{R}_{bafe}$

$$\mathcal{R}_g = \frac{l_g}{\mu_0 A_g} = \frac{5 \times 10^{-3}}{4\pi 10^{-7} \times 2 \times 2 \times 10^{-4}} = 9.94 \times 10^6 \text{ At/Wb}$$

$$\mathcal{R}_{be(core)} = \frac{l_{be(core)}}{\mu_c A_c} = \frac{51.5 \times 10^{-2}}{1200 \times 4\pi 10^{-7} \times 4 \times 10^{-4}} = 0.82 \times 10^6 \text{ At/Wb}$$

The loop equations are

$$\Phi_1(\mathcal{R}_{bafe} + \mathcal{R}_{be} + \mathcal{R}_g) + \Phi_2(\mathcal{R}_{be} + \mathcal{R}_g) = F_1$$

$$\Phi_1(\mathcal{R}_{be} + \mathcal{R}_g) + \Phi_2(\mathcal{R}_{bcde} + \mathcal{R}_{be} + \mathcal{R}_g) = F_2$$

or

$$\Phi_1(13.34 \times 10^6) + \Phi_2(10.76 \times 10^6) = 5000$$

$$\Phi_1(10.76 \times 10^6) + \Phi_2(13.34 \times 10^6) = 5000$$

or

$$\Phi_1 = \Phi_2 = 2.067 \times 10^{-4} \text{ Wb}$$

The air gap flux is

$$\Phi_g = \Phi_1 + \Phi_2 = 4.134 \times 10^{-4} \text{ Wb}$$

The air gap flux density is

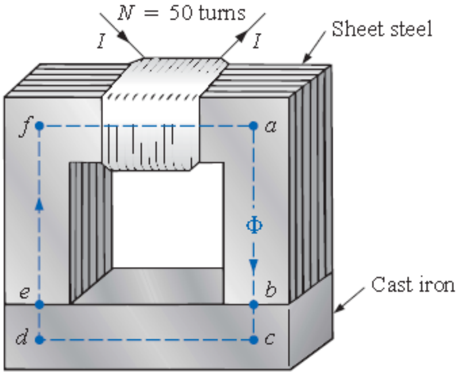
$$B_g = \frac{\Phi_g}{A_g} = \frac{4.134 \times 10^{-4}}{4 \times 10^{-4}} = 1.034 \text{ T}$$

The magnetic intensity in the air gap is

$$H_g = \frac{B_g}{\mu_0} = \frac{1.034}{4\pi 10^{-7}} = 0.822 \times 10^6 \text{ At/m}$$

<u>Example (8):</u>

The electromagnet shown in figure below has picked up a section of cast iron. Determine the current I required to establish the indicated flux in the core assuming that the relative permeability of sheet steel is 6165 and for cast iron is 2662.



$l_{ab} = l_{cd} = l_{ef} = l_{fa} = 4$ in.
$l_{bc} = l_{de} = 0.5$ in.
Area (throughout) $= 1$ in.^{2}
$\Phi = 3.5 \times 10^{-4}$ Wb

First, we must first convert to the metric system. However, since the area is the same throughout, we can determine the length for each material rather than work with the individual sections:

l_{efab} = 4 in. + 4 in. + 4 in. = 12 in. = 12×0.0254 = 0.3048 m

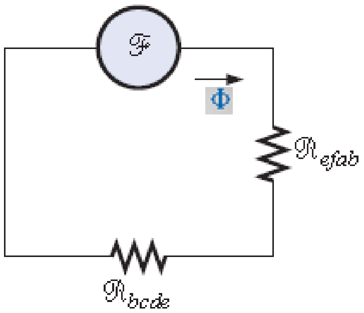
l_{bcde} = 0.5 in. + 4 in. + 0.5 in. = 5 in × 0.0254 = 0.127 m

The C.S.A = 1× $(0.0254)^2$ = 6.452×10^{-4} m^2

Now we need to calculate the reluctance for each section:

$$\Re_{efab} = \frac{l_{efab}}{\mu_{steel}\ A} = \frac{0.3048}{6165 \times 4\pi \times 10^{-7} \times 6.452 \times 10^{-4}} = 60978.62945$$

$$\Re_{bcde} = \frac{l_{bcde}}{\mu_{iron}\ A} = \frac{0.127}{2662 \times 4\pi \times 10^{-4} \times 6.452 \times 10^{-4}} = 58842.54486$$

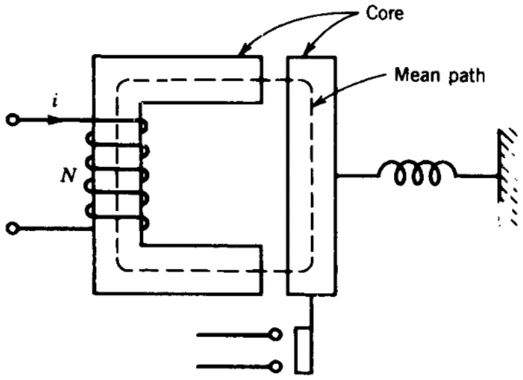


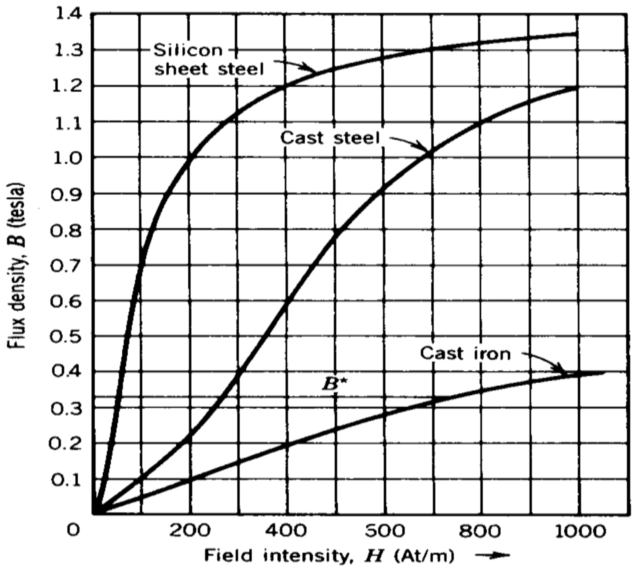
$50\times I = \Phi\times(\Re_{efab} + \Re_{bcde}) \rightarrow I = 0.839$ A

Example (9):

For the cast-steel magnetic circuit relay with N = 500 and the mean path is 360 mm and the air gap lengths are 1.5 mm each. A flux density of 0.8 T is required to actiuate the relay. Using the B-H curve of the cast steel core shown below, find:

a) The current in the coil,
b) The relative permeability of the core,
c) If the air gaps become zero, calculate the required current in the coil.





$$B_c = 0.8\ \text{T}, \qquad H_c = 510\ \text{At/m}$$

$$\text{mmf } F_c = H_c l_c = 510 \times 0.36 = 184\ \text{At}$$

For the air gap,

$$\text{mmf } F_g = H_g 2l_g = \frac{B_g}{\mu_0} 2l_g = \frac{0.8}{4\pi 10^{-7}} \times 2 \times 1.5 \times 10^{-3}$$

$$= 1910\ \text{At}$$

Total mmf required:

$$F = F_c + F_g = 184 + 1910 = 2094\ \text{At}$$

Current required:

$$i = \frac{F}{N} = \frac{2094}{500} = 4.19\ \text{amps}$$

(b) Permeability of core:

$$\mu_c = \frac{B_c}{H_c} = \frac{0.8}{510} = 1.57 \times 10^{-3}$$

Relative permeability of core:

$$\mu_r = \frac{\mu_c}{\mu_0} = \frac{1.57 \times 10^{-3}}{4\pi 10^{-7}} = 1250$$

(c)

$$F = H_c l_c = 510 \times 0.36 = 184\ \text{At}$$

$$i = \frac{184}{500} = 0.368\ \text{A}$$

Note that if the air gap is not present, a much smaller current is required to establish the same flux density in the magnetic circuit.

Example (10):

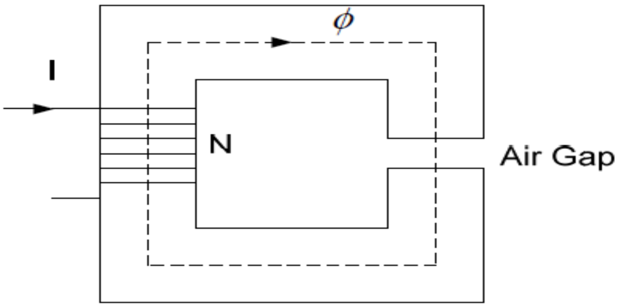
The magnetic circuit shown below has the following dimensions: A_c= 16 cm^2, l = 40cm, l_g = 0.5mm and N = 350 turns. The core is made of a material with the B-H relationship given below. For B = 1.0 T in the core, find:

a) The flux φ and the total flux linkage λ, where λ=N φ.

b) The required current to set this flux if there is no air gap.

c) The required current with the presence of an air gap.

B (Tesla)	H (A.T)
0.6	12.5
0.8	15.0
1.0	20.0
1.2	31.0
1.4	55.0



a) $\phi = BA_c = 1.0\times16\times10^{-4} = 1.6\,mWb$

$\lambda = N\phi = 350\times1.6\times10^{-3} = 0.56\,Wb.t$

b) With no air-gap

$F = \Re_c\phi = NI$

$\therefore I = \dfrac{\Re_c\phi}{N}$

$\Re_c = \dfrac{l}{\mu_c A_c},$

$\mu_c = \dfrac{B}{H} = \dfrac{1.0}{20.0} = 0.05$

$\Re_c = \dfrac{40\times10^{-2}}{0.05\times16\times10^{-4}} = 5000\ At/wb$

$\therefore I = \dfrac{5000\times1.6\times10^{-3}}{350} = 22.86\ mA$

c) With air-gap

$$F = NI = (\Re_c + \Re_g)\phi$$

$$\Re_c = \frac{l_c - l_g}{\mu_c A_c} \cong 5000,$$

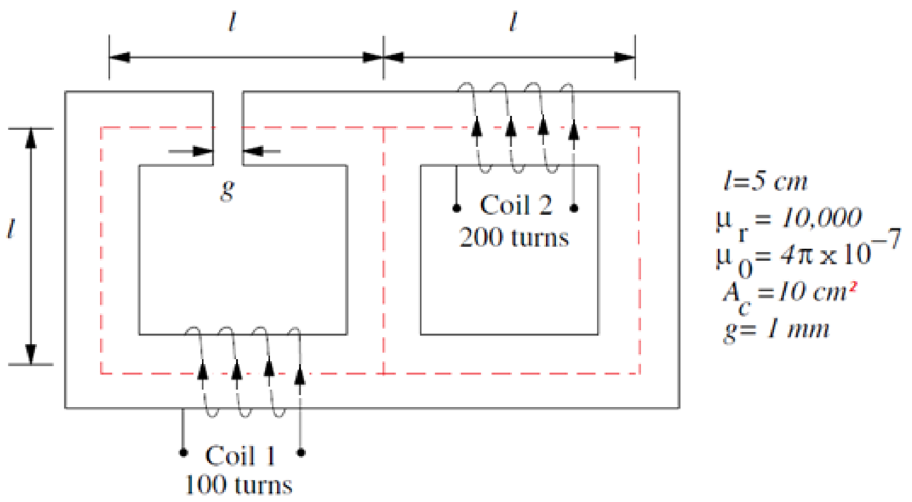
$$\Re_g = \frac{l_g}{\mu_g A_g} = \frac{0.5\times10^{-3}}{(4\pi\times10^{-7})\times16\times10^{-4}} = 248{,}679.6$$

$$I = \frac{(\Re_c + \Re_g)\phi}{N} = 1.16\ A$$

In this example, the current needed to set the same flux in case of magnetic circuits with air gap compared to those circuits without air-gap is much higher.

Example (11):

Two coils are wound on a magnetic core with an air-gap as shown in figure below. Find all of the magnetic fluxes in this magnetic system, assuming that the applied electric currents $i_1 = 4$A and $i_2 = 2$A.



It is convenient to think of the flux contour as consisting of several parts of different reluctances. Let R_1 denote the lump reluctance associated with parts (a) and (b) of the magnetic circuit. The length of the contour representing this part of the magnetic system is $l_1 = l + l + (l - g) = 3l - g = 14.9$ cm

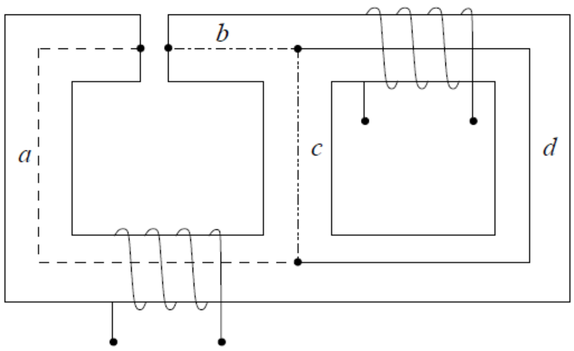
$$\mathcal{R}_1 = \frac{3l - g}{\mu A_c}$$

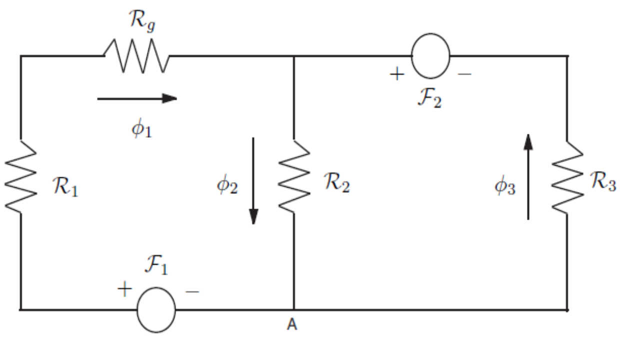
Similarily, we calculate the reluctances R_2, R_3 of parts (c) and (d) of the magnetic circuit as well as the reluctance R_g of the air gap:

$$\mathcal{R}_2 = \frac{l}{\mu A_c}, \quad \mathcal{R}_3 = \frac{3l}{\mu A_c}, \quad \mathcal{R}_g = \frac{g}{\mu_0 A_c}$$

$$\mathcal{R}_1 = 1.1857 \times 10^4,$$
$$\mathcal{R}_2 = 3.9789 \times 10^3,$$
$$\mathcal{R}_3 = 1.1937 \times 10^4,$$
$$\mathcal{R}_g = 7.9577 \times 10^5,$$

$F_1 = 100 \times 4 = 400$ At and $F_2 = 200 \times 2 = 400$ At





Applying KVL for the loop $AF_1R_1R_gR_2A$:

$F_1 = (R_1+R_g)\ \varphi_1 + R_2\ \varphi_2$

Applying KVL for the loop A $R_2F_2R_3A$:

$F_2 = R_3\ \varphi_3 + R_2\ \varphi_2$

Applying KCL at node A: $\varphi_3 + \varphi_1 = \varphi_2$

Substituting in the above two equations:

$F_1 = (R_1+R_g)\ \varphi_1 + R_2\ (\varphi_3 + \varphi_1) = (R_1+R_g+R_2)\ \varphi_1 + R_2\ \varphi_3$

$400 = 811605.9\ \varphi_1 + 3978.9\ \varphi_3$ -----(1)

$F_2 = R_3\ \varphi_3 + R_2\ (\varphi_3 + \varphi_1) = R_2\ \varphi_1 + (R_2+R_3)\ \varphi_3$

$400 = 3978.9\ \varphi_1 + 15915.9\ \varphi_3$ -----(2)

Solving the above two equations:

$0 = 807627\ \varphi_1 - 11937\ \varphi_3$

$\varphi_1 = 0.01478\ \varphi_3$ -----(3)

Substituting with this value in (2)

$400 = 3978.9\ (0.01478\ \varphi_3) + 15915.9\ \varphi_3 = 15974.7081\ \varphi_3$

Then $\varphi_3 = 25.0396$ mWb

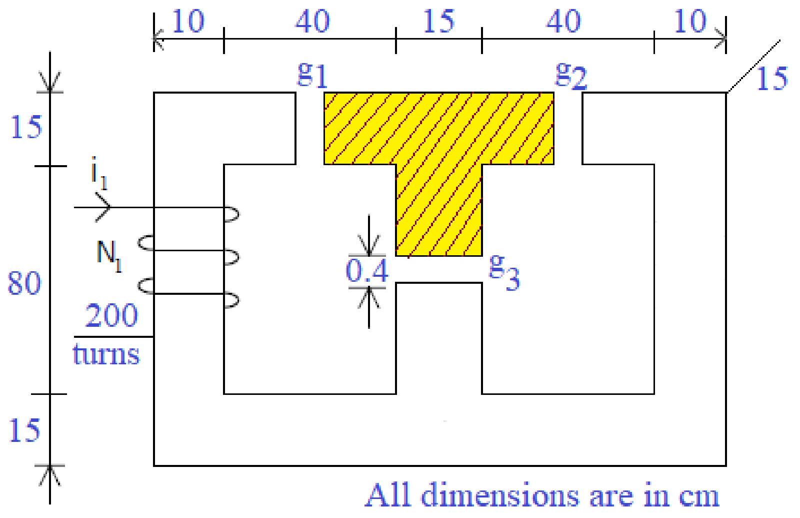
Substituting with the value of φ_3 in (3)

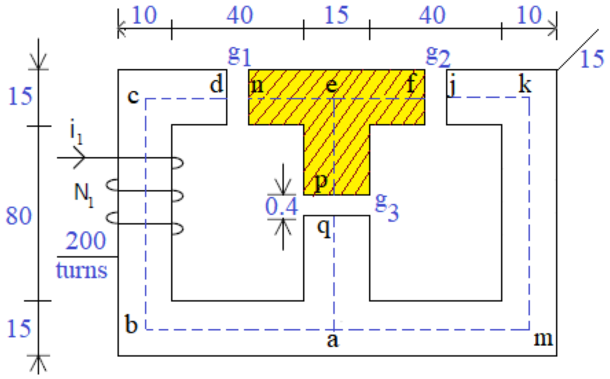
Then $\varphi_1 = 0.37$ mWb

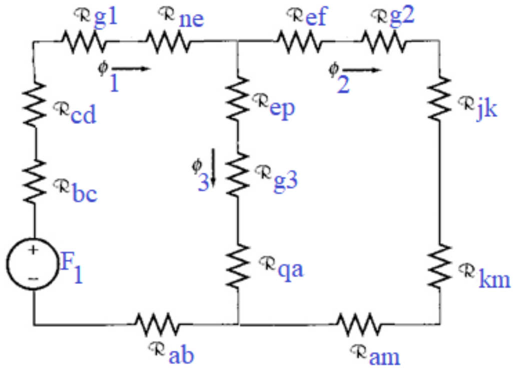
Since $\varphi_2 = \varphi_1 + \varphi_3 = 25.4096$ mWb

Example (12):

The magnetic circuit shown below has three identical air gaps (g_1, g_2 and g_3) and one coil with 200 turns. The circuit is made of two different materials (shaded material has a relative permeability of 1800 but the clear one has a relative permeability of 3000). Calculate the current (i_1) to be passed in the coil (N_1) to establish a flux density of 0.6 T along the air gap (g_3). Consider the fringing effect at the air gap (g_3) by 7% and the fringing effect is ignored at the other air gaps.







Reluctance Calculation:

$$\Re_{AB} = \Re_{AM} = \frac{l}{\mu\, A} = \frac{52.5 \times 10^{-2}}{3000 \times 4\pi \times 10^{-7} \times 15 \times 15 \times 10^{-4}} = 6189.3589\ A.t/Wb$$

$$\Re_{BC} = \Re_{KM} = \frac{l}{\mu\, A} = \frac{95 \times 10^{-2}}{3000 \times 4\pi \times 10^{-7} \times 10 \times 15 \times 10^{-4}} = 16799.6884\ A.t/Wb$$

$$\Re_{CD} = \Re_{JK} = \frac{l}{\mu\, A} = \frac{24.8 \times 10^{-2}}{3000 \times 4\pi \times 10^{-7} \times 15 \times 15 \times 10^{-4}} = 2923.7353\ A.t/Wb$$

$$\Re_{NE} = \Re_{EF} = \frac{l}{\mu\, A} = \frac{27.3 \times 10^{-2}}{1800 \times 4\pi \times 10^{-7} \times 15 \times 15 \times 10^{-4}} = 5364.11105\ A.t/Wb$$

$$\Re_{EP} = \frac{l}{\mu\, A} = \frac{47.3 \times 10^{-2}}{1800 \times 4\pi \times 10^{-7} \times 15 \times 15 \times 10^{-4}} = 9293.8627\ A.t/Wb$$

$$\Re_{QA} = \frac{l}{\mu\, A} = \frac{47.3 \times 10^{-2}}{3000 \times 4\pi \times 10^{-7} \times 15 \times 15 \times 10^{-4}} = 5576.3176\ A.t/Wb$$

The reluctance of air gaps

$$\Re_{G1} = \Re_{G2} = \frac{l_g}{\mu_o\, A} = \frac{0.4 \times 10^{-2}}{4\pi \times 10^{-7} \times 15 \times 15 \times 10^{-4}} = 141471.0605\ A.t/Wb$$

$$\Re_{G3} = \frac{l_{g3}}{\mu_o\, A} = \frac{0.4 \times 10^{-2}}{4\pi \times 10^{-7} \times 1.07 \times 15 \times 15 \times 10^{-4}} = 132215.9444\ A.t/Wb$$

Since the flux density along the air gap $G_3 = 0.6$ T, then

$$\varphi_3 = B \times A = 0.6 \times (15 \times 15 \times 10^{-4} \times 1.07) = 0.014445\ Wb$$

The MMF through the central leg (F_3)

$$F_3 = \varphi_3 \times (\Re_{EP} + \Re_{G3} + \Re_{QA}) = 0.014445 \times 147086.1247 = 2124.6591\ A.t$$

The MMF through the Right-Hand leg (F_2)

$$F_2 = \varphi_2 \times (\Re_{EF} + \Re_{G2} + \Re_{JK} + \Re_{KM} + \Re_{AM}) = F_3 = 2124.6591\ A.t$$

$$\varphi_2 \times (172747.9542) = 2124.6591\ A.t$$

$$\varphi_2 = 0.0123\ Wb$$

But $\Phi_1 = \Phi_2 + \Phi_3$

$$\varphi_1 = 0.014445 + 0.0123 = 0.026744\ Wb$$

The MMF through the Left-Hand leg (F_1)

$$F_1 = 200 \times I_1 = \varphi_1 \times (\Re_{AB} + \Re_{BC} + \Re_{CD} + \Re_{G1} + \Re_{NE}) + F_3$$

$$200 \times I_1 = 0.026744 \times (172747.9542) + 2124.6591 = 6744.6624$$

$$I_1 = 33.7233\ A$$

Magnetic Behavior of Ferromagnetic Materials:

Materials which are classified as non-magnetic show a linear relationship between the flux density B and coil current I. In other words, they have constant permeability. Thus, for example, in free space, the permeability is constant. But in iron and other ferromagnetic materials permeability is not constant. For magnetic materials, a much larger value of B is produced in these materials than in free space. Therefore, the permeability of magnetic materials is much higher than μ_o. However, the permeability is not linear anymore but does depend on the current over a wide range. Thus, the permeability is the property of a medium that determines its magnetic characteristics. In other words, the concept of magnetic permeability corresponds to the ability of the material to permit the flow of magnetic flux through it. In electrical machines and electromechanical devices, a somewhat linear relationship between B and I is desired, which is normally approached by limiting the current.

Look at the magnetization curve and B-H curve shown in Fig. 16, where the flux density (B) produced in the core is plotted versus the flux intensity (H) producing it. This plot is called a **magnetization curve.** At first, a small increase in H produces a huge increase in B. After a certain point, further increases in H produce relatively smaller increases in B. Finally, there will be no change at all as you increase H further. The region in which the curve flattens out is called saturation region, and the core is said to be **saturated**. The region where B changes rapidly is called **the unsaturated region**. The transition region is called the 'knee' of the curve.

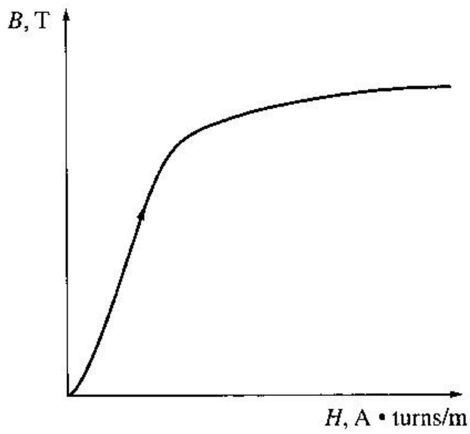


Fig. 16, B-H curve for ferromagnetic materials

Therefore, if the resulting flux has to be proportional to the mmf, then the core must be operated in the unsaturated region. Generators and motors depend on magnetic flux to produce voltage and torque, so they need as much flux as possible. So, they operate near the knee of the magnetization curve (flux not linearly related to the mmf). This non-linearity as a result gives peculiar behaviors to machines. As magnetizing intensity H increased, the relative permeability first increases and then starts to decrease.

Energy Losses in a Ferromagnetic Core:

1) **Hysteresis loss**

If an AC current flows through a coil, we expect that during the positive cycle there is a relation between B and H as we discussed in the previous section. On the other hand, during the negative cycle, the B-H relation is a mirror to that obtained in positive cycle. The curve would be as shown in Fig. 17.

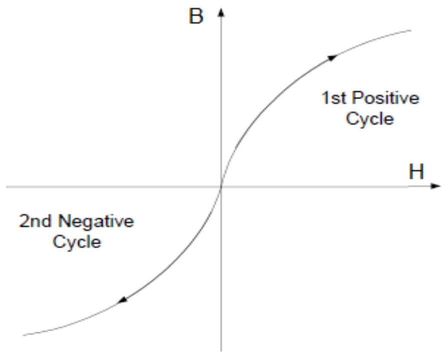
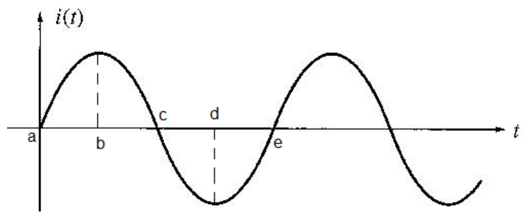


Fig. 17, Theoretical AC magnetic behavior

Unfortunately, the above assumption is only correct provided that the core is 'perfect' i.e. there are no residual flux present during the negative cycle of the ac current flow. A typical flux behavior (or known as hysteresis loop) in a ferromagnetic core is as shown in Fig. 18. HYSTERESIS is the dependence on the preceding flux history and the resulting failure to retrace flux paths.

The explanation to that curve is that, when we apply AC current (assuming flux in the core is initially zero), as current increases, the flux traces the path *ab*. (saturation curve). When the current decreases, the flux traces out a different path from the one when the current increases. When current decreases, the flux traces out path *bcd*. When the current increases again, it traces out path *deb*. From these paths we noted that:

- When MMF is removed, the flux does not go to zero (**residual flux)**. This is how permanent magnets are produced.
- To force the flux to zero, an amount of mmf known as **coercive MMF** must be applied in the opposite direction.



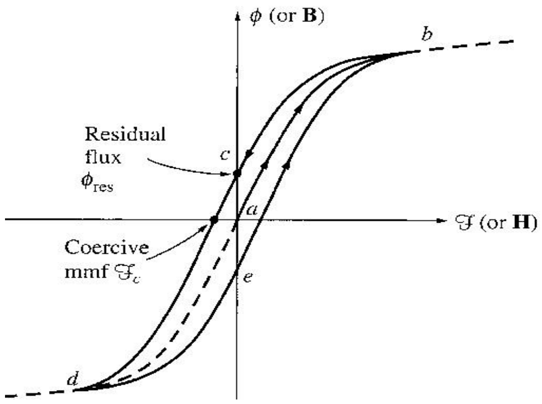


Fig. 18, Hysteresis path

This related to the atoms of iron and similar metals like cobalt, nickel, and some of their alloys tend to have their magnetic fields closely aligned with each other. Within the metal, there is an existence of small regions known as **domains** where in each domain there is a presence of a small magnetic field which randomly aligned through the metal structure such that the net magnetic field is zero, as shown in Fig. 19.

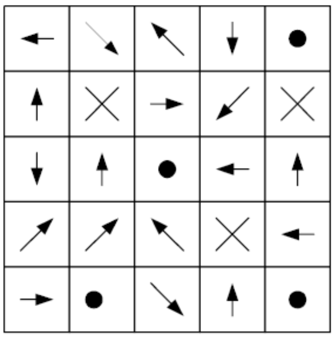


Fig. 19, Magnetic domain orientation in a metal before applying a magnetic field.

When mmf is applied to the core, each magnetic field will align with respect to the direction of the magnetic field. That explains the exponential increase of magnetic flux during the early stage of magnetisation. As more and more domain are aligned to the magnetic field, the total magnetic flux will maintain at a constant level hence as shown in the magnetisation curve (saturation). When mmf is removed, the magnetic field in each domain **will try** to revert to its random state. However, **not all** magnetic field domain's would revert to its random state hence it remained in its previous magnetic field position. This is due to the lack of energy required to disturb the magnetic field alignment.

Hence the material will retain some of its magnetic properties (permanent magnet) up until an external energy is applied to the material. Therefore, in an AC current situation, to realign the magnetic field in each domain during the opposite cycle would require extra mmf (known as coercive mmf). This extra energy requirement is known as **hysteresis loss**. The larger the material, the more energy is required hence the higher the hysteresis loss. Area enclosed in the hysteresis loop formed by applying an AC current to the core is directly proportional to the energy lost in a given ac cycle.

2) Eddy-current loss

At first, the changing flux induces voltage within a ferromagnetic core. This voltage cause swirls of current to flow within the core called eddy currents. Energy is dissipated (in the form of heat) because these eddy currents are flowing in a resistive material (iron). The amount of energy lost to eddy currents is proportional to the **size of the paths** they follow within the core. To reduce energy loss, ferromagnetic core should be broken up into small strips, or laminations, and build the core up out of these strips. An insulating oxide or resin is used between the strips, so that the current paths for eddy currents are limited to small areas as shown in Fig. 20.

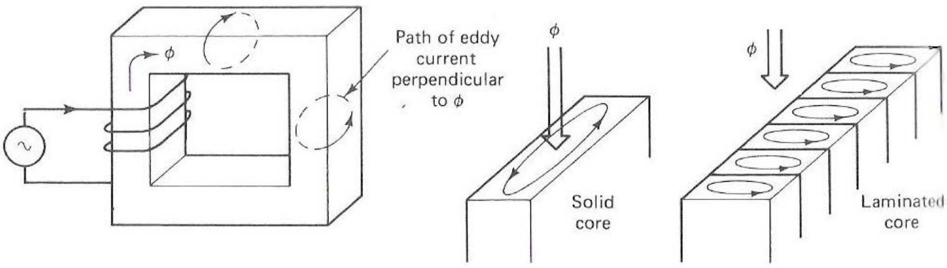


Fig. 20, Eddy-current in ferromagnetic materials

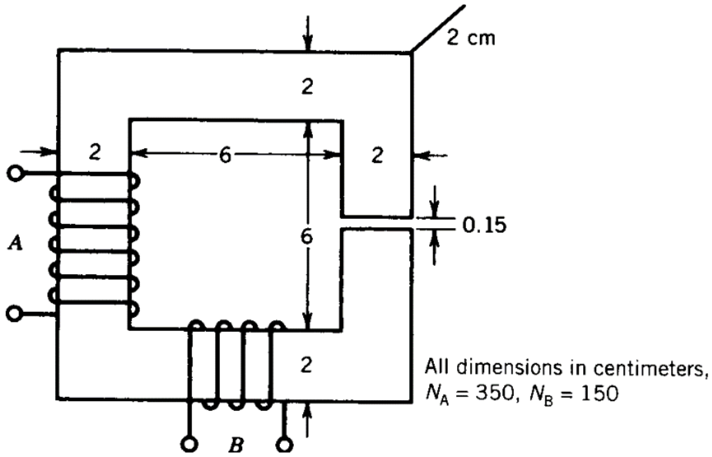
SHEET (1) Magnetic Circuits

Problem (1)

An inductor is made of two coils, A and B, having 350 and 150 turns, respectively. The coils are wound on a cast steel core and in directions as shown The two coils are connected *in series* to a dc voltage.

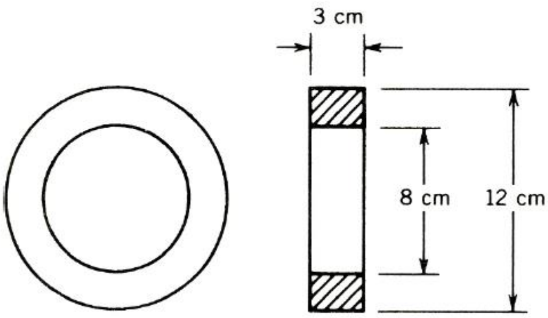
(a) Determine the two possible values of current required in the coils to establish a flux density of 0.5 T in the air gap.

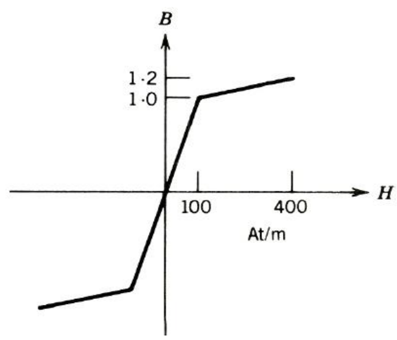
(b) If coil B is now disconnected and the current in coil A is adjusted to 2.0 A, determine the mean flux density in the air gap.



Problem (2)

A toroidal core has a rectangular cross section as shown. It is wound with a coil having 100 turns. The *B–H* characteristic of the core may be represented by the linearized magnetization curve shown.





Determine the minimum value of the current for which the complete core has a flux density of 1.0 Wb/m^2 or greater.

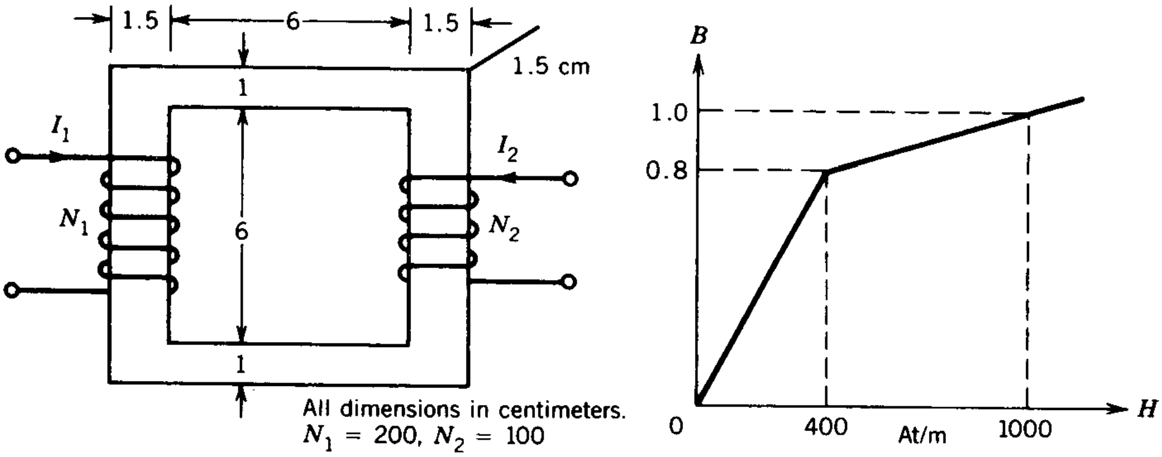
Problem (3)

For the magnetic circuit shown below and the B−H curve for the core material can be approximated as two straight lines as given.

(a) If $I_1 = 2.0$ A, calculate the value of I_2 required to produce a flux density of 0.6 T in the vertical limbs.

(b) If $I_1 = 0.5$ A and $I_2 = 1.96$ A, calculate the total flux in the core.

Neglect magnetic leakage.



Problem (4)

For the series-parallel magnetic circuit that made of sheet steel with relative permeability of 4000 as shown in Fig. 1. There is a small gap of 0.002 m in the structure of the left limb. Assume that fringing in the air gap increases the effective CSA of the gap by 5%. Find the value of the current (I) required to establish a flux in the gap of $\Phi_1 = 2 \times 10^{-4}$ Wb.

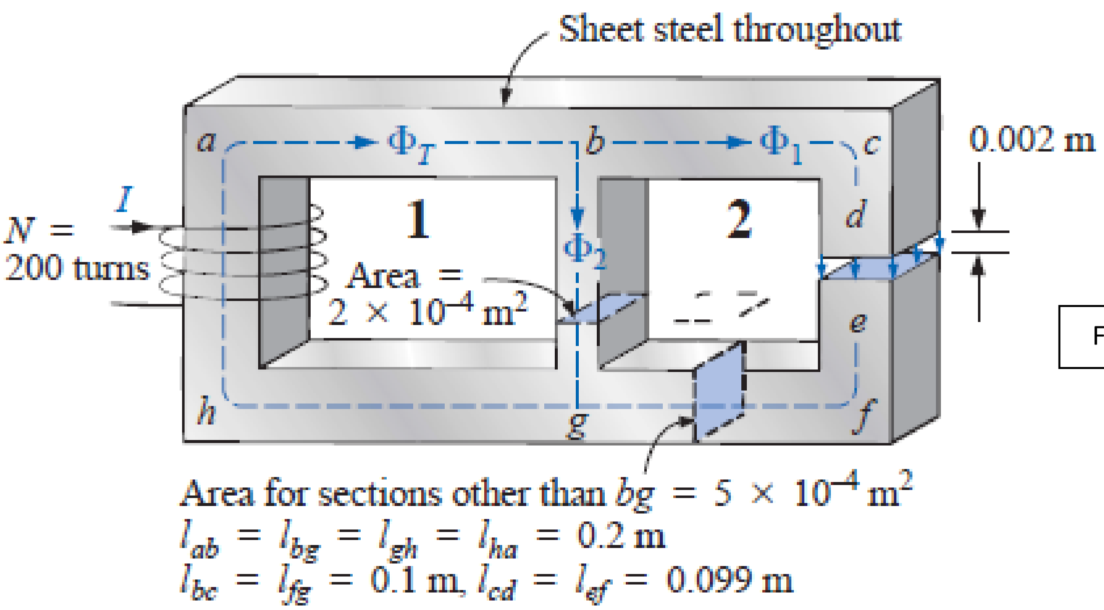
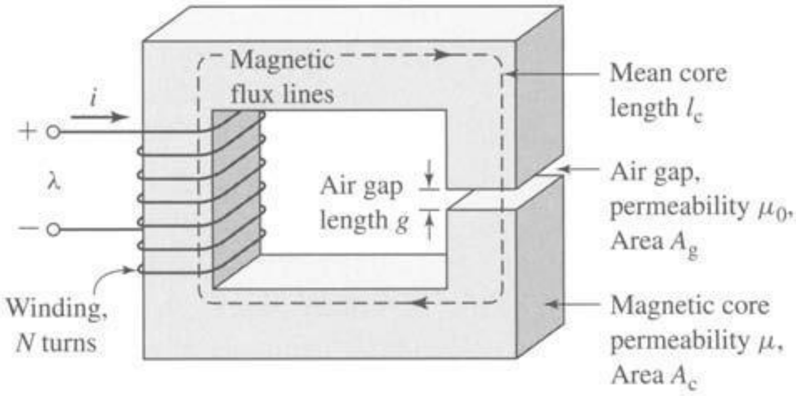


Fig. (1)

Problem (5)

The magnetic circuit shown below has dimensions Ac = Ag = 9 cm^2, g = 0.050 cm, *l*c = 30 cm, and N = 500 tums. Assume the value μ_r = 70,000 for core material. For the condition that the magnetic circuit is operating with $B_c = 1.0$ T, find

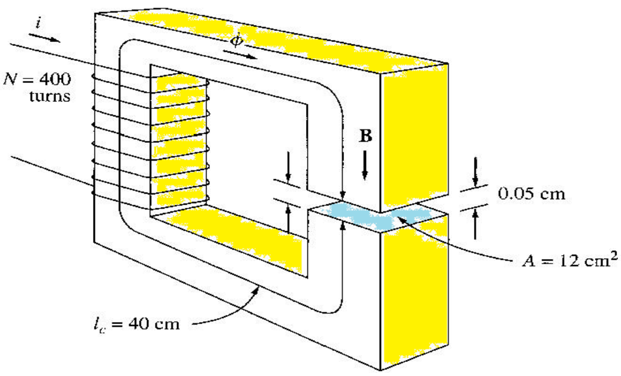
(a) the reluctances R_c and R_g.
(b) the flux φ and the current i.



Problem (6)

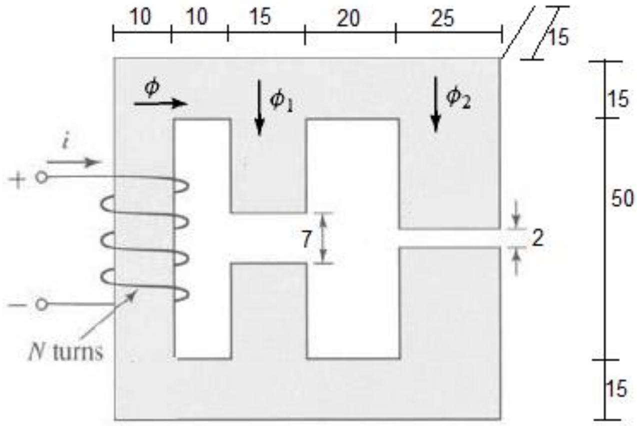
A ferromagnetic core with mean path length is 40cm. There is a small gap of 0.05cm in the structure of the otherwise whole core. The CSA of the core is 12cm^2, the relative permeability of the core is 4000, and the coil of wire on the core has 400 turns. Assume that fringing in the air gap increases the effective CSA of the gap by 5%. Find (a) The total reluctance of the flux path (iron plus air gap)

(b) The current required to produce a flux density of 0.5T in the air gap.



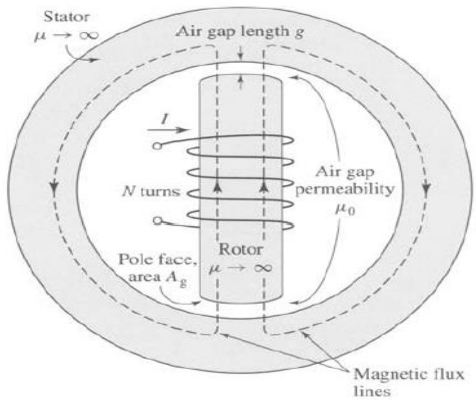
Problem (7)

A ferromagnetic core with two air gaps with dimensions in centimeter indicated below, the relative permeability of the core is 5000, and the coil of wire on the core has 400 turns and draws a current of 10A. Find the main flux ϕ and the flux in each leg ϕ1 and ϕ2.



Problem (8)

The magnetic structure of a synchronous machine is shown schematically in Figure below. Assuming that rotor and stator iron have infinite permeability (μ→∞), find the air-gap flux and flux density. Consider I = 10 A, N = 1000 turns, g = 1 cm, and A_g=2000 cm^2.



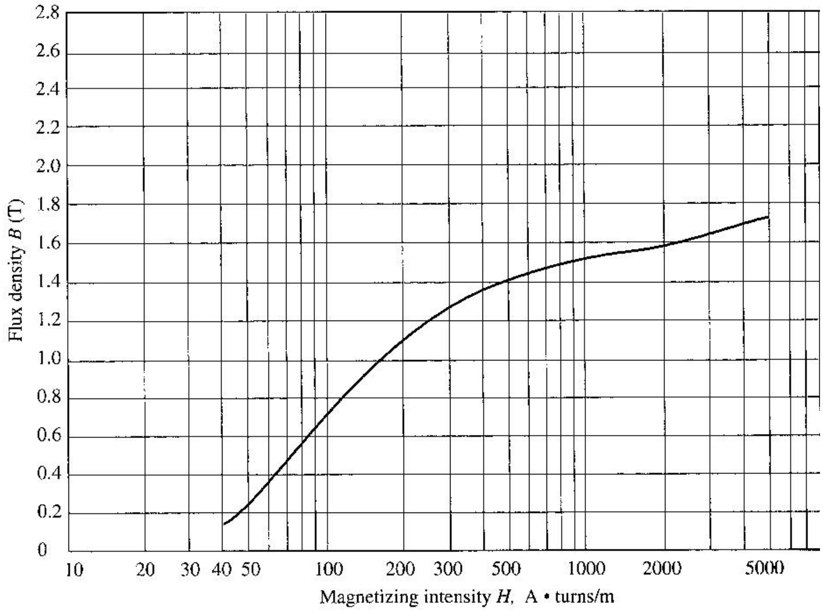
Problem (9)

A square magnetic core has a mean path length of 55cm and a CSA of 150cm^2. A 200 turn coil of wire is wrapped around one leg of the core. The core is made of a material having the magnetization curve shown below. Find:

a) How much current is required to produce 0.012 Wb of flux in the core?

b) What is the core's relative permeability at that current level?

c) What is its reluctance?


2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
Flux density B (T)
10
20
30
40
50
100
200
300
500
1000
2000
5000
Magnetizing intensity H, A • turns/m